Fourier Transforms

Many engineering problems lead to ordinary or partial differential equations which have to be solved under various types of conditions formulated from the problem. We are already familiar with the solution of higher order ordinary differential equations with initial conditions (initial value problems) using Laplace transforms. Solution of some partial differential equations with boundary conditions (boundary value problems) can be obtained with the help of Fourier transforms. Fourier transform is a mathematical tool for representing signals in time domain and frequency domain. It converts a time domain signal to frequency domain. It is an extension of Fourier series to non-periodic signals. Fourier transform is used in communications engineering as well as image and signal processing.

Definition: The Fourier Transform of *f(t)* or complex Fourier transform of is defined as

The inverse Fourier transform of is given by,

**Special Case of Laplace transform:**

By the definition of bilateral Laplace transform

where .

If we set we obtain

which is the Fourier transform of

Properties of Fourier Transform:

1. Linearity property: If and are any 2 constants and , are any 2 functions whose Fourier transforms exist, then

Proof: By definition of Fourier transform

1. Change of scale property: If is any non-zero real constant and then

Proof:

Let

If and if

1. Shifting property: If is a real constant then
2. Time shifting property:

Proof:

Let

If and if

1. Frequency shifting property

Proof:

1. Time reversal property: If is the Fourier transform of then the Fourier transform of is

Proof:

Using the transformation

1. Similarly,

Fourier transform of basic functions:

1. Unit impulse function,

Impulse in time constant in frequency

1. Constant function,

By Duality property, we have

If then

We know that

( impulse function is even)

Hence constant in time Impulse in frequency

By frequency shifting property, we have

Also

1. Trigonometric functions, and
2. Unit step function,

Unit step function is defined as



and its Fourier transform is

Problems:

1. Obtain the Fourier transform of given by

Solution: Fourier transform of is given by

for and otherwise.\

2. Obtain the Fourier transform of given by

Solution: Fourier transform of is given by

3. Obtain the Fourier transform of given by

Sol:

4. Obtain the Fourier transform of

Sol: Fourier transform of is given by

Here

5. Obtain the Fourier transform of

Sol: Fourier transform of is given by

Here

Applying Bernoulli’s rule to each of the integral

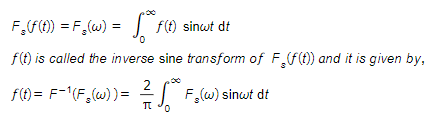
6. Obtain the Fourier transform of , where is positive constant

Solution: Fourier transform of is given by

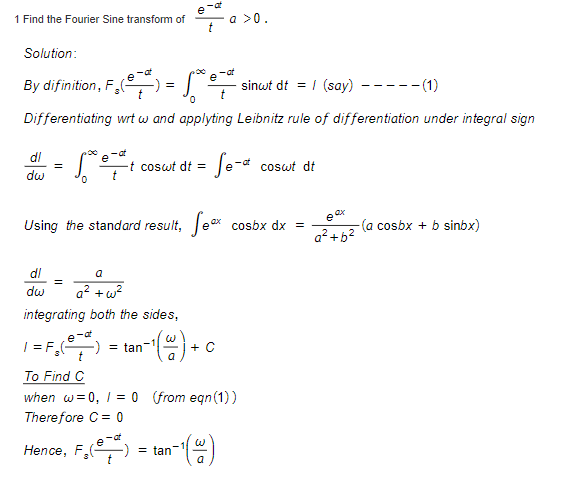
Since for by data we have

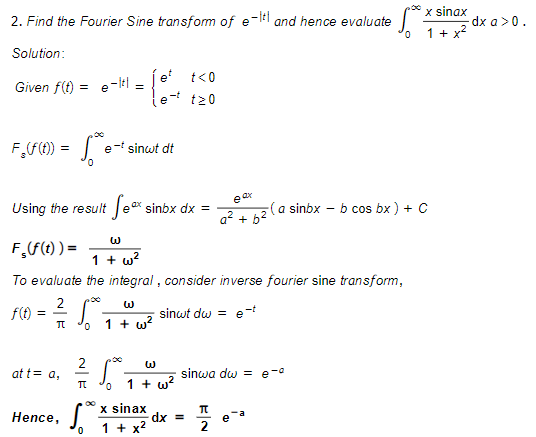
**FOURIER SINE TRANSFORMS**

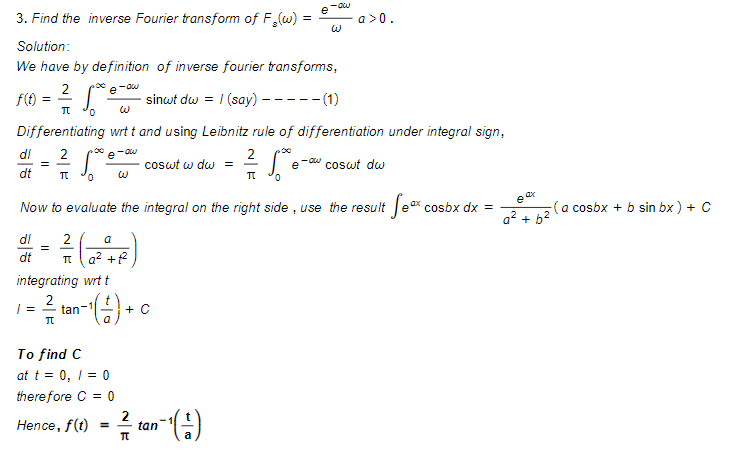
Fourier Sine transform of f(t) t>0, is given by



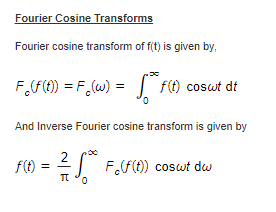
**Problems**

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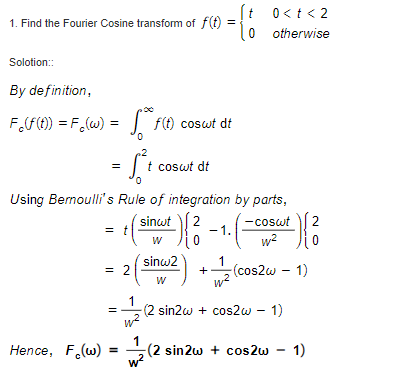
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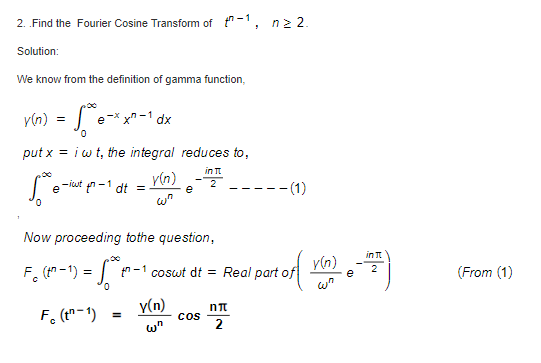
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**FOURIER COSINE TRANSFORMS**

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**Problems:**

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